

Fracture Mechanics for Crack Propagation in Drying Soils

P.C. Prat¹, A. Ledesma, M.R. Lakshmikantha, H. Levatti and J. Tapia
Technical University of Catalonia, Barcelona, Spain.

ABSTRACT: Fracture parameters (Fracture toughness and Tensile strength) are determined experimentally at different moisture contents. The tensile strength was determined using existing equipment (direct method) at the Soil Mechanics Laboratory of UPC whereas new equipment was designed for the fracture toughness determination. The results of tensile strength tests are consistent with published literature. Fracture toughness decreases as the moisture content increases. A numerical model is presented for crack initiation and propagation in drying soils. The model combines the concepts of soil mechanics and fracture mechanics. Crack initiation and propagation criteria are based on fracture parameters. Experimental values of fracture parameters are used to trigger the crack initiation and propagation.

Keywords: soil cracks, fracture mechanics, size-effect, H-M model

1 Introduction

Fracture mechanics has been widely used in recent years in the study of the mechanical behaviour of brittle or quasi-brittle materials such as concrete, rock, ceramics, etc. Its application to soils, however, has been scarce and with limited scope. Natural soils with water contents close to its liquid limit have a soft consistency, much different from that of quasi-brittle materials, which have practically zero tensile strength. In such a material, of course fracture mechanics cannot be applied. However, once the soil dries and becomes partially saturated, suction increases rapidly changing dramatically the consistency: the soil becomes much harder and its tensile strength develops to reach values that cannot be neglected. In fact, the soil becomes quasi-brittle.

During the process of drying, the soil cracks. Cracking initiates shortly before the soil reaches a near-solid quasi-brittle consistency. Although crack initiation can be explained by classical Soil Mechanics effective stress theory (Abu-Hejleh and Znidarcic 1995; Lloret et al. 1998; Kodikara et al. 2000; Rodríguez 2006; Rodríguez et al. 2007), crack development and propagation appears to be energy-driven, and therefore Fracture Mechanics is an adequate framework (Harison and Hardin 1994; Harison et al. 1994; Lima and Grismer 1994; Vallejo 1994; Hallett et al. 1995; Hallett and Newson 1998; Hallett and Newson 2001; Prat et al. 2002; Ávila 2004; Hallett and Newson 2005). A recent experimental campaign conducted by the authors (Prat et al. 2006, Lakshmikantha et al., 2007a) indicates that indeed there is considerable energy-related size effect, a prominent feature of fracture mechanics.

Further research on this topic required the determination of the fracture mechanics properties of the drying soil: fracture energy, fracture toughness and tensile strength. Fracture and tensile tests on compact soil specimens of the same material and with consistencies as close as possible to the specimens used in the previous campaign have been conducted to determine these properties, which are later used to calibrate the size-effect formula. In this paper we present the results of the laboratory experiments with the determination of the fracture parameters, and we show how fracture mechanics can be used as an adequate framework to describe and predict the process of crack formation and propagation when the soil dries due to changes of environmental conditions. A coupled H-M finite element based numerical model, being developed to study this type of problems, is also discussed.

¹ Corresponding author — Dept. of Geotechnical Engineering and Geosciences UPC; Jordi Girona 1-3, 08034 Barcelona, Spain. Phone: +34 934016511; fax: +34 934017251; e-mail: pere.prat@upc.edu

2 Experimental determination of fracture parameters

2.1 Fracture toughness – CT tests

The equipment for the determination of fracture toughness (K_{IC}) was developed at the Soil Mechanics Laboratory of UPC, using the equipment design of Ávila (Ávila 2004). Test specimens of two different sizes were tested (Table 1) at different moisture content, with a constant density $\gamma = 1.95 \pm 0.05 \text{ kN/m}^3$. Figure 1 show the schematics of the equipment and a photograph of the setup

The soil used in the experiments is a Barcelona silty clay collected from a construction site near the laboratory at a depth of approximately 4 m below ground surface. This type of soil is commonly found in the area and has been extensively studied in the past in the laboratory, its geotechnical and hydro-mechanical properties being well known (Barrera 2002). Its main characteristics are: unit weight of soil particles $\gamma_s = 27.1 \text{ kN/m}^3$; liquid limit $w_L = 32\%$; plastic limit $w_p = 16\%$. According to the unified soil classification system, the soil can be classified as low plasticity clay (CL). The dry soil was sieved through a mechanical sieve of 1.18 mm (sieve no. 16); the material passing was used for the test. Distilled water was added in required quantity to achieve the intended moisture content. Once a visibly homogeneous paste was obtained, its moisture content was determined before pouring it into the CT-moulds. Moisture content was determined again when the experiment was completed. The CT-mould was filled with the prepared material in three layers in order to have a homogeneous density. Loading pins were inserted to the specimens after removing from the moulds and a Methacrylate plate was inserted between the specimen and the nuts of the loading pin in order to ensure the correct load transmission to the right fracture zone just below the initial crack. The load was applied manually, with a constant frequency. The fracture load was determined counting all the weights in the loading pan after the specimen failed. The procedure was repeated for all the specimens.

The moisture contents of the test specimens were 16%, 18%, 19%, and 21%, with an initial crack length of 10, 15, and 20 mm for the medium and 20, 30, and 40 mm for the big specimen. For each size, moisture content and initial crack length, tests were repeated with a minimum of two specimens and in some cases with three. A total of 55 specimens were tested. Table 1 gives the details of the geometry of the test specimens, with length (L), width (B), and thickness (W). A circular hole of diameter (ϕ) was made from a distance (d) to the edge of the specimen for loading pins.

$$K_I = \frac{P}{B\sqrt{D}} \hat{k}(\alpha) \quad (1)$$

$$G_{IC} = (1 - \nu^2) \frac{K_I^2}{E} \quad (2)$$

Fracture toughness (K) was calculated by eq.1, where D is the characteristic dimension of the specimen (in the present case $W = D$); P is the fracture load; and B is the width of the specimen. $\hat{k}(\alpha)$ is a function depending on the geometry of the specimen ($\alpha = a/W$). $\hat{k}(\alpha)$ was calculated using two different empirical formulas, given by Eq. 3 (ASTM-E399-83 1983) and Eq. 4 (Srawley 1976). The fracture energy (G) was calculated using Eq. 2, with $\nu = 0.3$ and $E = 4.2 \text{ MPa}$ (Barrera 2002).

$$k(\alpha) = (30.96\alpha - 195.8\alpha^2 + 730.6\alpha^3 - 1186.3\alpha^4 + 754.6\alpha^5) \quad (3)$$

$$k(\alpha) = (2 + \alpha) \left[\frac{0.886 + 4.64\alpha - 13.32\alpha^2 + 14.72\alpha^3 - 5.6\alpha^4}{(1 - \alpha)^{3/2}} \right] \quad (4)$$

Table 1. Details of CT-test specimens

| Mould | L (mm) | B (mm) | W (mm) | d (mm) | ϕ (mm) |
|--------|--------|--------|--------|--------|-------------|
| medium | 60 | 25 | 45 | 15 | 12 |
| big | 120 | 50 | 90 | 30 | 24 |

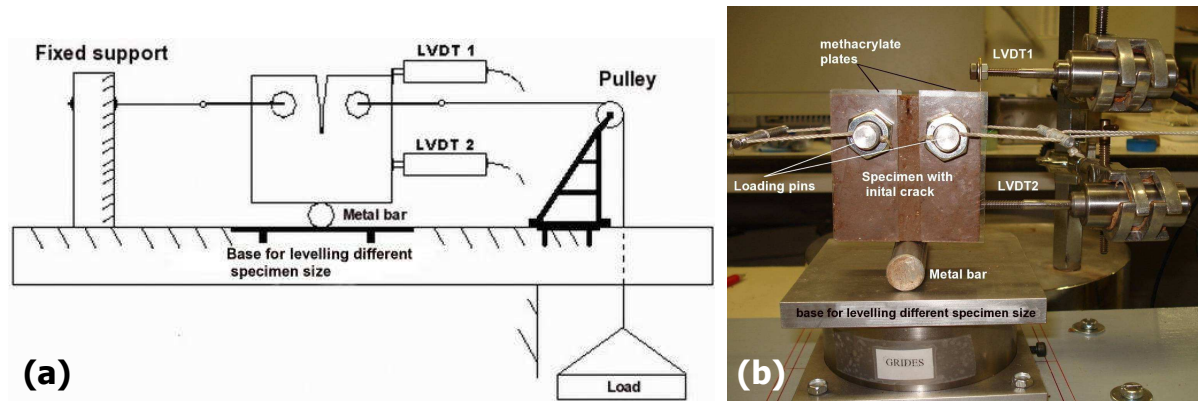


Figure 1. (a) Schematic diagram of CT-test equipment; (b) photograph of the setup.

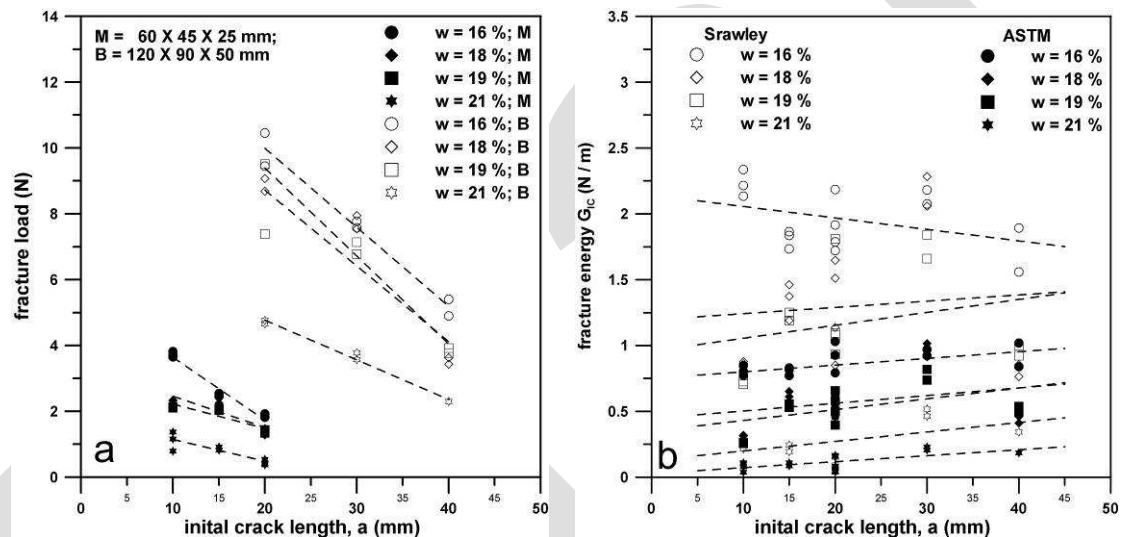


Figure 2: (a) Fracture load (b) Fracture energy

Figure 2a shows the variation of fracture load for the two specimen's sizes (medium and big) at various moisture contents. As a common and well known trend, here also the fracture load increases with decrease in initial crack length (Lee et al. 1988; Nichols and Grismer 1997) irrespective of specimen size and moisture content. Other observed important behaviour, particular to soils, is the effect of moisture content on the fracture load: fracture load decreases as the moisture content of the test specimens increases for both sizes's tested.

Figure 2b shows the variation of the fracture energy, G , with the initial crack length. The regression lines calculated with the ASTM and Srawley methods show that G is approximately constant (Lee et al. 1988) for a given moisture content, proving that G is a material property (depending on moisture content for soils).

2.2 Tensile strength – Direct method

Tensile strength was determined using an equipment designed by Rodríguez (2002), the equipment is similar to the one explained by Mikulish and Gudeus (1995). A similar experiment has been reported by Nahlawi et al. (2004). The equipment used in the present work is made up of 3 main parts (see Figure 3a): two pieces of trapezoidal shape, one fixed and another one freely movable on application of external force, and a central part that is removed just before the application of the load; this is the only part of the specimen which will be subjected to tension during the test. Figure 3b shows a photograph of one specimen in the tensile apparatus.

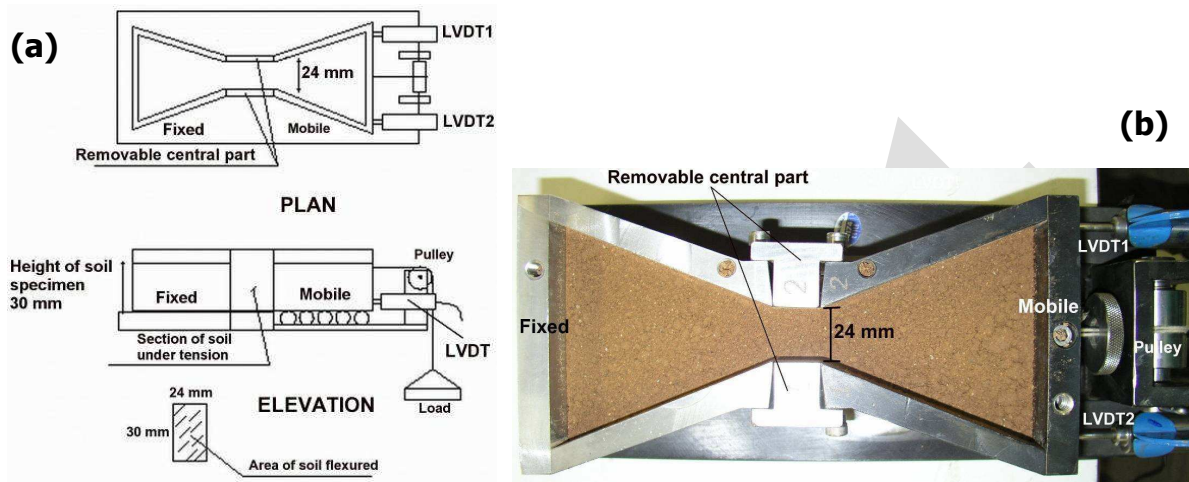


Figure 3. (a) Schematic diagram of the direct tensile strength equipment and (b) photograph of one sample in the apparatus

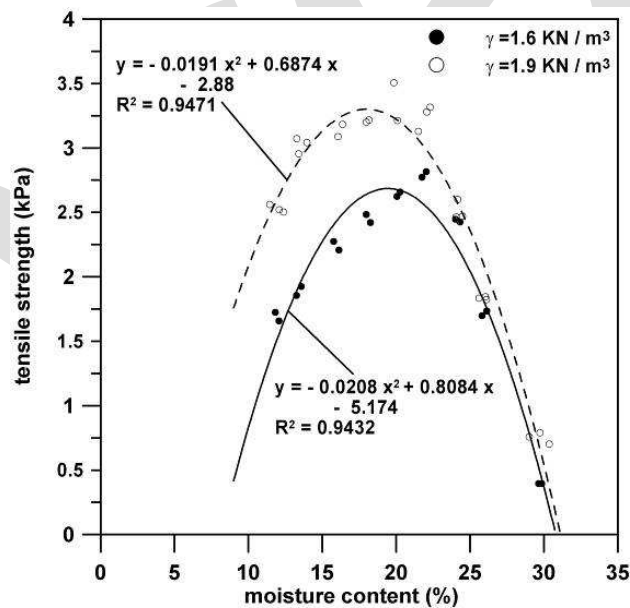


Figure 4: Variation of tensile strength with moisture content

A total of 42 tests were conducted for two different densities (18 tests with $\gamma = 1.6 \text{ kN/m}^3$, and 24 tests with $\gamma = 1.9 \text{ kN/m}^3$) with average moisture content ranging from 12% to 30%. For each density and moisture content the tests were repeated with a minimum of two specimens and in some cases three. The soil used and the preparation of the material was the same as explained earlier for the fracture toughness tests. The depth of the soil placed in the equipment was fixed and the weight of the soil was varied to obtain different densities. The tensile strength (σ_T) was calculated directly by dividing the area of soil under tension by the total load applied. Figure 4 shows the variation of tensile strength with moisture content for two dry densities. A clear difference in the tensile strength for different densities on the dry-side is observed, whereas on the wet-side the difference is smaller. Similar behaviour has been observed by other authors (Favaretti 1996; Tamarakar et al. 2005; Rodríguez 2006).

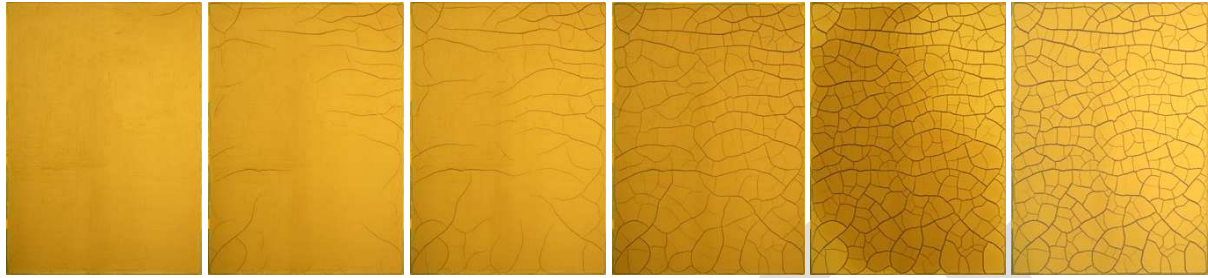


Figure 5. Sequence showing crack development from appearance of first crack to the completion of drying and crack pattern formation corresponding to test A0-20.

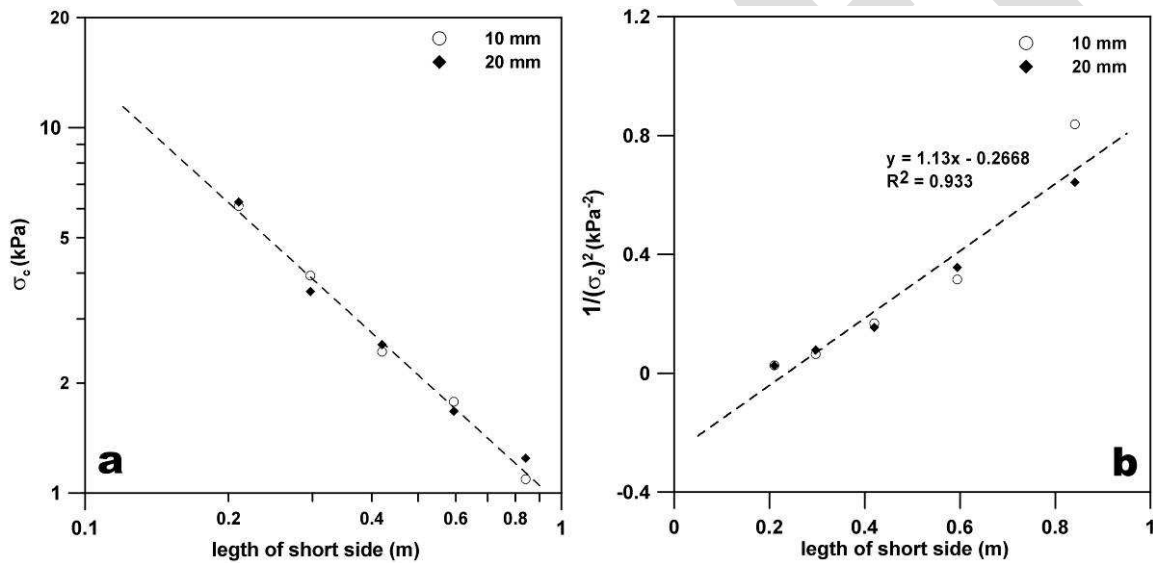


Figure 6: Size effect plots: (a) cracking stress decrease with increasing size; (b) linear interpolation to obtain Bažant's size effect law's parameters

3 Why LEFM?

Soils are particulate media which inherently have fissures and crack-like imperfections; these flaws can trigger fracture or initiate a crack due to environmental action or when loaded. Plastic or nonlinear strains unavoidably prevail in the vicinity of these flaws or crack tip. Although LEFM holds good for linear-elastic materials only, it does provide an ideal and simple way of estimating the amount of energy required to create a new free surface in the material. As an example, the results of an experimental study (Prat et al. 2006, Lakshmikantha et al., 2007a) conducted to investigate size effect to ascertain whether Fracture Mechanics plays a significant role in the process of formation and/or propagation of cracks during drying of soils due to changes in environmental conditions such as temperature or humidity will be summarized here. More details about the tests can be found in the previous references since because space restrictions cannot be fully provided here.

The experiments consist of five geometrically similar specimens (sizes DIN A0 to A4) with two thicknesses for each size (10 and 20 mm), subjected to drying conditions in a laboratory controlled environment. Figure 5 shows the developing crack pattern for the largest specimen (A0, 20 mm). Similar sequences are observed for the other sizes and thicknesses. The main parameters that describe the evolving crack pattern (crack density factor, crack length, intersection angles, etc.) are obtained using image analysis techniques performed on digital high resolution photographs of the specimens taken throughout the tests (Lakshmikantha et al., 2007b). With these results it can be proved, in a qualitative manner, that cracking stress depends on the size of the specimen and therefore that fracture mechanics can be used to model these phenomena. Figure 6 shows the main results of the analysis. Figure 6a represents the size effect law, where the cracking stress decreases as the size of the specimen increases. Figure 6b is the regression plot used to calibrate Bažant's size-effect law. A recent work (Ávila 2004) also shows evidence of size effect in the cracking of drying soils, and that the results as well follow Bažant's size effect law (Bažant 1984b).

4 Model

4.1 Hydro-Mechanical model

Two state stress variables are adopted in this model: net stress σ^* and suction s .

$$\sigma^* = \sigma - p^a \mathbf{1} \quad (5)$$

$$s = p^a - p \quad (6)$$

With the assumption that atmospheric pressure p^a is constant and equal to zero, the state variables become the total stress σ and the negative pore water pressure p .

$$\sigma^* = \sigma \quad \text{and} \quad s = -p \quad (7)$$

Under the assumptions of small-strain theory, isothermal equilibrium and negligible inertial forces, we obtain the following balance equations. First, the linear momentum balance equation for a two phase medium, where ρ is the density and \mathbf{g} is the gravity vector.

$$\text{div } \mathbf{g} + \rho \mathbf{g} = 0 \quad (8)$$

Second, the mass balance equation for water, where ρ^w is the water density, \mathbf{q} is Darcy's velocity, n the porosity and S_r the degree of saturation.

$$\text{div}(\rho^w \mathbf{q}) + \frac{\partial}{\partial t}(\rho^w n S_r) = 0 \quad (9)$$

We can summarize the coupled problem through the next system of differential equations:

$$\begin{cases} \mathbf{K} \frac{\partial \bar{\mathbf{u}}}{\partial t} + \mathbf{Q} \frac{\partial \bar{p}}{\partial t} - \mathbf{f}^u = \mathbf{0} \\ \mathbf{P} \frac{\partial \bar{\mathbf{u}}}{\partial t} + \mathbf{S} \frac{\partial \bar{p}}{\partial t} + \mathbf{H} \bar{p} - \mathbf{f}^p = \mathbf{0} \end{cases} \quad (10)$$

Where $\bar{\mathbf{u}}$ and \bar{p} are respectively nodal displacements and nodal pore pressure. \mathbf{K} , \mathbf{Q} , \mathbf{P} , \mathbf{S} and \mathbf{H} are matrices, and \mathbf{f}^u and \mathbf{f}^p vectors, that result from the FEM approach.

The algebraic system of equations that results from this formulation is highly nonlinear and non-symmetric, in general. For this reason we need to use iterative strategies to solve it. The first equation in (10) is the mechanical part and the second one is the hydraulic part.

4.2 Unsaturated soil mechanics concepts

For the hydro-mechanical formulation we need to employ two constitutive models. First we apply a mechanical constitutive model based on the concept of state surfaces. Next a hydraulic constitutive model, including unsaturated flow (Darcy's law) is used. We write both constitutive models as follow:

$$\varepsilon_v = -\frac{e}{1+e_0} = a_1 \ln(\sigma + a_4) + a_2 \ln\left(\frac{p + p_{ref}}{p_{ref}}\right) + a_3 \left[\ln(\sigma + a_4) \ln\left(\frac{p + p_{ref}}{p_{ref}}\right) \right] \quad (11)$$

$$\mathbf{q} = -\mathbf{K}(S_r) \cdot (\nabla p - \rho^w \mathbf{g}) \quad (12)$$

Where ε_v is the volumetric strain, e and e_0 are the current and initial void ratios, a_1, a_2, a_3, a_4 are state surface constants and p_{ref} is a reference pressure.

Furthermore, we need the relation between suction and degree of saturation. In our case we chose the Van Genuchten equation (Van-Genuchten 1980) [13]:

$$S_r = \left[1 + \left(\frac{p}{P_0 f_n} \right)^{\frac{1}{1-\lambda}} \right]^{-\lambda} \quad (13)$$

In (13) S_r is the degree of saturation, λ is a material parameter, P_0 is the air entry value at the reference porosity n_0 and f_n is a function of the porosity and of the material parameter η .

4.3 Fracture parameters

Tensile strength of the soil is used for the crack initiation (Lee et al. 1988), whereas for crack propagation concepts of LEFM (Atkinson 1991) are used. This section explains how the conditions of stress state in a drying soil obtained from the H-M model are used for the crack initiation and propagation.

4.3.1 Crack initiation

Drying soil can be considered as a continuum media before the crack initiation, therefore applying a tensile strength criterion whereby computed principal stresses (from H-M model) are compared with an experimentally determined tensile strength (σ_T) to predict the crack initiation.

4.3.2 Crack propagation and direction

Considering only Mode-I failure and applying the $\sigma(\theta)_{\max}$ theory, the criterion for crack propagation can be expressed as,

$$\sigma(\theta)_{\max} \sqrt{2\pi r} \geq K_{IC} \quad (14)$$

where, $\sigma(\theta)_{\max}$ is obtained from the H-M model and (K_I) is experimentally determined, and r is an arbitrary value that depends on the type of element used in the FEM formulations. The value of r is 2/3 of the height of the element for triangular elements.

The direction of crack propagation is in a plane normal to the direction of greatest tension, i.e., at θ_0 such that $\tau_{r\theta} = 0$.

$$\sigma(\theta)_{\max} \sqrt{2\pi r} = \cos\left(\frac{\theta_0}{2}\right) \left[K_I \cos^2\left(\frac{\theta_0}{2}\right) \right] \quad (15)$$

5 Conclusions

Fracture parameters determined experimentally are in agreement with the existing published data. Fracture parameters were determined at different moisture contents; these values are more realistic when used in any numerical model for cracking in drying soils. LEFM seems to be the most simple fracture mechanics theory to explain cracking in soils. This is due to ease with which the LEFM parameters determination. Experimental evidences of existence of Size-effect further strengthens the applicability of LEFM. The proposed model combining soil mechanics and fracture mechanics provides an ideal frame work for numerical solution of cracking in drying soils.

Acknowledgements

The research reported in this paper has been carried out within the framework of a research project financed by the Spanish Ministry of Education and Science (CGL2006-09847). Their support is gratefully acknowledged.

References:

- Abu-Hejleh, A. N. and Znidarcic, D. (1995). Desiccation theory for soft cohesive soils. ASCE J.Geotech.Engrg. **121**(6):493-502.
- ASTM-E399-83 (1983). Standard test method for plane-strain fracture toughness of metallic materials. American Society for Testing and Materials.

- Atkinson, B. K. (1991). Fracture mechanics of rock. London, Academic Press.
- Ávila, G. (2004). Estudio de la retracción y el agrietamiento de arcillas. Aplicación a la arcilla de Bogotá (in Spanish). PhD Thesis, Dept. of Geotechnical Engineering and Geosciences, Technical University of Catalonia, Barcelona, Spain.
- Barrera, M. B. (2002). Estudio experimental del comportamiento hidro-mecánico de suelos colapsables (in Spanish). PhD Thesis, Dept. of Geotechnical Engineering and Geosciences, Technical University of Catalonia, Barcelona, Spain.
- Bazant, Z. P. (1984). Size effect in blunt fracture: Concrete, Rock, Metal. *ASCE J. Engrg. Mech.* **110**:518-535.
- Favaretti, M. (1996). Tensile strength of compacted clays. State of the art in Unsaturated Soils. E. E. Alonso and P. Delage, eds, Balkema, Rotterdam.
- Hallett, P. D., Dexter, A. R. and Seville, J. P. K. (1995). The application of fracture mechanics to crack propagation in dry soil. *European Journal of Soil Science* **46**(4):591-599.
- Hallett, P. D. and Newson, T. A. (1998). Desiccation of a sensitive clay: application of the model CRACK. Discussion. *Canadian Geotechnical Journal* **35**:1109-1110.
- Hallett, P. D. and Newson, T. A. (2001). A simple fracture mechanics approach for assessing ductile crack growth in soil. *Soil Sci. Soc. Am. J.* **65**:1083-1088.
- Hallett, P. D. and Newson, T. A. (2005). Describing soil crack formation using elastic-plastic fracture mechanics. *European Journal of Soil Science* **56**:31-38.
- Harison, J. A. and B. O. Hardin (1994). Cracking in clays – Solutions to problems in earth structures. *Int. J. Num. Anal. Methods in Geomechanics* **18**:467-484.
- Harison, J. A., Hardin, B. O. and Mahboub, K. (1994). Fracture toughness of compacted cohesive soils using ring test. *Journal of Geotechnical Engineering* **120**(5):872-891.
- Kodikara, J., Barbour, S. L. and Fredlund, D. G. (2000). Desiccation cracking in soils. Proc. of the Asian Conf. on unsaturated soils-Unsaturated soils for Asia, Rahardjo, Toll and Leong eds., Balkema, Rotterdam.
- Lakshmikantha, M.R., Prat, P.C., and Ledesma, A. (2007a). Experimental evidences of size-effect in soil cracking, Report GT-041, ETSECCPB-UPC, Barcelona.
- Lakshmikantha, M.R., Prat, P.C., and Ledesma, A. (2007b). Real-time evaluation of a desiccating soil and 2-D crack network quantification, Report GT-040, ETSECCPB-UPC, Barcelona
- Lee, F. H., Lo, K. W. and Lee, S. L. (1988). Tension crack development in soils. *ASCE J. Geotech. Engrg.* **114**(8): 915-929.
- Lima, L. A. and Grismer, M. E. (1994). Application of fracture mechanics to cracking of saline soils. *Soil Science* **158**(2):86-96.
- Lloret, A., Ledesma, A., Rodríguez, R. L., Sánchez, M. J., Olivella, S. and Suriol, J. (1998). Crack initiation in drying soils. 2nd Int. Conference on Unsaturated Soils, Beijing China, International Academic Publisher.
- Mikulish, W. A. and Gudeus, G. (1995). Uniaxial tension, biaxial loading and wetting tests on loess. First Int. Conf. on Unsaturated Soils, Paris, Balkema/Presses des Ponts et Chaussées.
- Nahlawi, H., Chakrabarti, S. and Kodikara, J.K. (2004). A direct tensile testing method for unsaturated materials. *Geotechnical Testing Journal, ASTM*, **27**(4):1-6.
- Nichols, J. R. and Grismer, M. E. (1997). Measurement of fracture mechanics parameters in silty-clay soils. *Soil Science* **162**(5):309-322.
- Prat, P. C., Ledesma, A. and Cabeza, L. (2002). Drying and cracking of soils: numerical modeling. 8th Int. Conference on Numerical Models in Geomechanics NUMOG VIII, Rome, Italy.
- Prat, P. C., Ledesma, A. and Lakshmikantha, M. R. (2006). Size effect in the cracking of drying soil. 16th European Conference of Fracture, Alexandroupolis, Greece.
- Rodríguez, R. (2002). Estudio experimental de flujo y transporte de cromo, níquel y manganeso en residuos de la zona minera de Moa (Cuba): Influencia del comportamiento hidromecánico(In Spanish). Barcelona, Technical University of Catalonia.
- Rodríguez, R. (2006). Hydrogeotechnical characterization of a metallurgical waste. *Canadian Geotechnical Journal*, **43**:1042-1060.
- Rodríguez, R., Sánchez, M.J., Ledesma, A. and Lloret, A. (2007). Experimental and numerical analysis of desiccation of a mining waste. *Canadian Geotechnical Journal*, **44**:644-658.
- Srawley, J. E. (1976). Wide range stress intensity factor expressions for ASTM E-399 standard fracture toughness specimens. *Int. J. Fracture* **95**:475-476.
- Tamarakar, S. B., Toyosawa, Y., Mitachi, T. and Itoh, K. (2005). Tensile strength of compacted and natural soils using newly developed tensile strength measuring apparatus. *Soils and Foundations* **45**(6):103-110.
- Vallejo, L. E. (1994). Application of fracture mechanics to soils: An overview. *Fracture Mechanics Applied to Geotechnical Engineering, ASCE Geotechnical Special Publication* **43**:1-20.
- Van-Genuchten, M. T. (1980). Closed-form equation for predicting the hydraulic conductivity of unsaturated soils. *Soil Science Society of America Journal* **44**(5):892-898.